# MIMO Receiver:

## Zero Forcing (ZF) Receiver

$$\bar{y} = H\bar{x} + \bar{w}$$

$\bar{y}$: $r \times 1$

$H$: $r \times t$

$\bar{x}$: $t \times 1$

$\bar{w}$: $r \times 1$

How to recover Transmit vector $\bar{x}$ at the receiver.

# Simple Scenario:

$r = t$

No. of Rx antennas = No. of Tx antennas.

$H$ —— $r \times t$

$t \times t$

→ Square Matrix if $r = t$

IF $H$ is invertible,

$\bar{y} \approx H\bar{x}$ ← System of Linear Equations.

Simple Receiver →

$$\hat{x} = H^{-1}\bar{y}.$$

IF $r > t$

ie Number of Rx antennas > Number of Transmit antennas.

Then H is NOT a square matrix

$\Rightarrow$ inverse of H does not exist.

$$\bar{y} = H\bar{x}$$

$r \times t$

There are $r$ equations
$t$ unknown quantities.

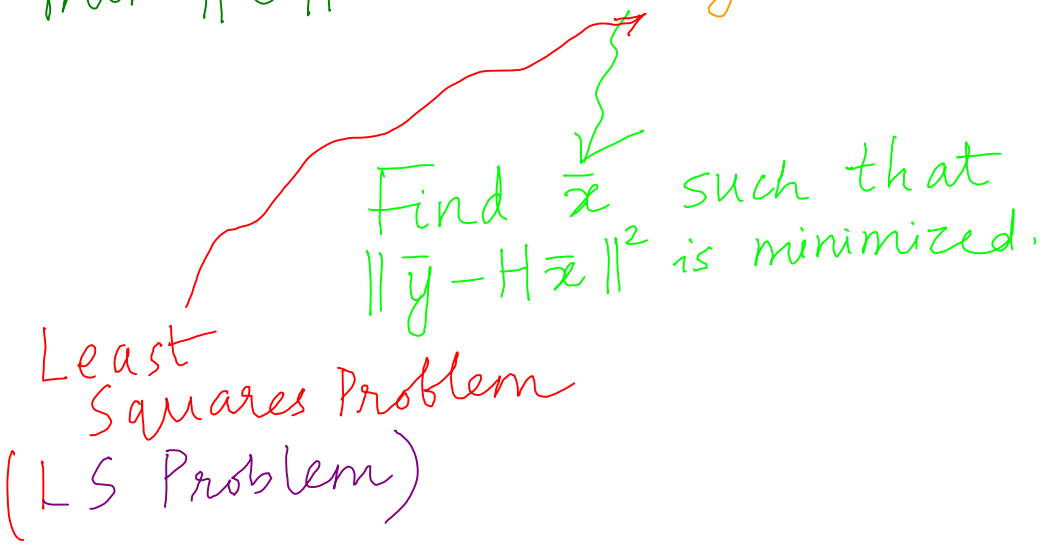
$$r > t$$

$\Rightarrow$ Number of Equations $>$ Number of unknowns.

$$\bar{e} = \bar{y} - H\bar{x}$$

To minize error,

$$\min \|\bar{e}\|^2 = \min \|\bar{y} - H\bar{x}\|^2$$



To find minimum, differentiate and set equal to zero.

However, $\bar{x}$ is a vector

$t \times 1$ Transmit vector

Therefore we have to differentiate wrto vector $\bar{x}$

Vector Derivative.

$F(\bar{x})$

function of vector $\bar{x}$

$$\frac{\partial F}{\partial \bar{x}} = \begin{bmatrix} \frac{\partial F}{\partial x_1} \\ \frac{\partial F}{\partial x_2} \\ \vdots \\ \frac{\partial F}{\partial x_t} \end{bmatrix}$$

Differentiate $F$ wrt to every element of $\bar{x}$.

For Example:

$$\bar{C}^T \bar{x} = \begin{bmatrix} C_1 & C_2 & \cdots & C_t \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_t \end{bmatrix}$$

$$= C_1 x_1 + C_2 x_2 + \cdots + C_t x_t$$

$$F(\bar{x}) = \bar{c}^T \bar{x}$$

$$\bar{c}^T \bar{x} = c_1 x_1 + c_2 x_2 + \cdots + c_t x_t$$

$$\frac{\partial(\bar{c}^T \bar{x})}{\partial x_1} = c_1$$

$$\frac{\partial(\bar{c}^T \bar{x})}{\partial x_2} = c_2$$

$$\vdots$$

$$\frac{\partial(\bar{c}^T \bar{x})}{\partial x_t} = c_t$$

$$\frac{\partial(\bar{c}^T \bar{x})}{\partial \bar{x}} = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_t \end{bmatrix} = \bar{c}$$

$$\bar{c}^T \bar{x} = \bar{x}^T \bar{c}$$

$$\frac{\partial(\bar{c}^T \bar{x})}{\partial \bar{x}} = \frac{\partial(\bar{x}^T \bar{c})}{\partial \bar{x}} = \bar{c}$$

$f(\bar{x}) = \bar{x}^T P \bar{x}$ ← where $P = P^T$

$$\frac{\partial(\bar{x}^T P \bar{x})}{\partial \bar{x}} = \frac{\partial(\bar{x}^T \underbrace{P\bar{x}}_{\bar{c}})}{\partial \bar{x}} + \frac{\partial(\underbrace{\bar{x}^T P}_{\bar{c}^T} \cdot \bar{x})}{\partial \bar{x}}$$

$$= P\bar{x} + (\bar{x}^T P)^T$$

$$= P\bar{x} + P^T \bar{x} = 2P\bar{x}$$

$$\frac{\partial(\bar{x}^T P \bar{x})}{\partial \bar{x}} = 2P\bar{x}$$

LS cost function

$$F(\bar{x}) = \|\bar{y} - H\bar{x}\|^2$$

$$= (\bar{y} - H\bar{x})^T (\bar{y} - H\bar{x})$$

$$= (\bar{y}^T - \bar{x}^T H^T)(\bar{y} - H\bar{x})$$

$$= \bar{y}^T\bar{y} - \bar{x}^T H^T \bar{y} - \bar{y}^T H\bar{x} + \bar{x}^T H^T H\bar{x}$$

$$= \bar{y}^T\bar{y} - 2\bar{x}^T H^T \bar{y} + \bar{x}^T H^T H\bar{x}$$

$$\|\bar{y} - H\bar{x}\|^2 = F(\bar{x}).$$

$$= \bar{y}^T\bar{y} - 2\bar{x}^T H^T\bar{y} + \bar{x}^T H^T H\bar{x}$$

$$\frac{\partial F(\bar{x})}{\partial \bar{x}} = 0 - 2H^T\bar{y} + 2H^T H\bar{x}$$

$$= 0$$

$$2H^T\bar{y} = H^T H\bar{x}$$

$$\boxed{\hat{x} = (H^T H)^{-1} H^T \bar{y}}$$

$$\hat{x} = (H^T H)^{-1} H^T \bar{y}$$

Zero Forcing Receiver

If H is complex

$$\hat{x} = (H^H H)^{-1} H^H \bar{y}$$

MIMO Zero Forcing (ZF) Receiver

$(H^H H)^{-1}$

$t \times r$ $\quad$ $r \times t$

$H^H H$ is $t \times t$

→ Square matrix

$(H^H H)^{-1} H^H$

Pseudo inverse of $H$.

$$(H^H H)^{-1} H^H \cdot H$$

$$= I$$

acting as left inverse of $H$.
Pseudo inverse.